3-Maruza. Tokli qo`zg`almas o`tkazgichli sistemasining energiyasi

Mavzuni o`zlashtirish uchun n qo`zg`almas o`tkazgichli (konturli) tok oqayotgan sistemaga kelayotgan energiyani aniqlash misolida ko`rib chiqamiz. Buning uchun tok $I_k$ qiymatining noldan qandaydir qiymatga oshgan yoki shu vaqtdagi oqim ilashimligi o`zgarishi davridagi energiyani ko`rib chiqib amalga oshirish mumkin. Qarshiligi $r_k$ bo`lgan k konturga tashqi EYUK (kuchlanish) u_k ulangan deb faraz qilaylik. Tok (oqim ilashimligi)ning o`zgarishi EYUK ning hosil bo`lishiga bog`liq, u holda zanjirning elektr balansi bizga ma`lum bo`lgan tenglama bilan aniqlanandi:

$$u_k + e_k = r_k i_k \tag{1.5a}$$

$$u_k = r_k i_k + \frac{d\psi_k}{dt}. \tag{1.5b}$$

(1.5b) tenglamaning chap va o`ng qismini $i_k$ ga ko`paytirib manba sarf qilayotgan quvvatni aniqlaymiz:

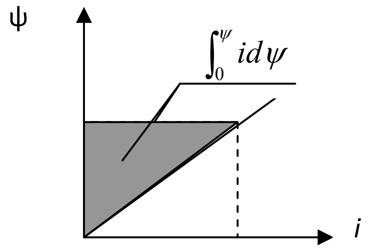
$$u_k i_k = r_k i_k^2 + i_k \frac{d\psi_k}{dt}. \tag{1.6}$$

(1.6) tenglamani dt ga ko`paytirib, berilgan vaqt intervali davo-mida k konturdagi manba bajargan ishni topamiz:

$$u_k i_k dt = dW_{элк} = r_k i_k^2 dt + i_k d\psi_k. \tag{1.7}$$

bu ishning bir qismi ($r_k i_k^2 dt$) issiqlik ko`rinishida yo`qoladi, qolgan qismi $(i_k d\psi_k)$ - konturning magnit maydoni ko`rinishida yig`iladi, ya`ni

$$dW_{магнк} = i_k d\psi_k$$



1.1- rasm.

SHunday qilib, n ta toklarning magnit energiyasi quyidagiga teng ekan:

$$W_{магн} = \sum_{k=1}^{n} \int_{0}^{\psi_k} i_k d\psi_k \ . \qquad (1.8)$$

Agarda i va ψ orasida chiziqli bog`lanish bo`lsa, (1.8) tenglamadagi integralning qiymatini 1.1-rasm yordamida topish mumkin.

CHiziqli sistemalar uchun (1.8) tenglamani konturining induktiv-ligi va o`zaro induktivligi orali yozish qulay. Malumki, k konturining oqim ilashimlig konturning barcha toklarining chiziqli funksiyasi hisoblanadi,ya`ni:

$$\psi_к = \sum_{k=1}^{n} L_{ks} \qquad (1.9a)$$

bu erda: L_{ks}-k va s konturlarning o`zaro induktivligi; L_{kk}=L_k-k konturning induktivligi.

(1.9a) tenglamani boshqacha ko`rinishda ham yozish mumkin:

$$\psi_к = L_k I_k + \sum_{\substack{s=1 \\ k \neq s}}^{n} L_{ks} I_s \ . \qquad (1.9b)$$

Bundan shu narsa ko`rinadiki, k kontur oqim ilashimligining o`zga-rishini quyidagicha yozish mumkin:

$$\Delta\psi_{\kappa} = L_k \Delta I_k + \sum_{s=1}^{n} L_{ks} \Delta I_s .$$

Demak, (1.8) tenglamaning integral osti qismini quyidagicha yozish mumkin:

$$\Delta W_{\text{магк}} = I_k \Delta\psi_k = I_k (L_k \Delta I_k + \sum_{s=1}^{n} L_{ks} \Delta I_s) . \tag{1.10}$$

yoki konturlar bo`yicha qo`shishni hisobga olgan holda (oqim ilashimligi bo`yicha integralsiz):

$$\Delta W_{\text{маг}} = \sum_{k=1}^{n} I_k (L_k \Delta I_k + \sum_{s=1}^{n} L_{ks} \Delta I_s). \tag{1.11}$$

Bu tegnlamani keyingi o`zgartirish uslubi tushintirish uchun ikki konturning yakka holatini ko`rib chiqamiz. Ular uchun (1.11) teng-lama quyidagi ko`rinishni egallaydi:

$$\Delta W_{\text{маг}} = L_1 I_1 \Delta I_1 + L_{12} I_1 \Delta I_2 + L_2 I_2 \Delta I_2 + L_{21} I_2 \Delta I_1 .$$

$L_{12}=L_{21}$ ekanligini hisobga olgan holda quyidgiga ega bo`lamiz:

$$\frac{\Delta W_{\text{маг}}}{\Delta t} = \frac{\Delta}{\Delta t}\left(\frac{1}{2} L_1 I_1^2\right) + \frac{\Delta}{\Delta t}\left(\frac{1}{2} L_2 I_2^2\right) + \frac{\Delta}{\Delta t}\left(L_{12} I_1 I_2\right)$$

yoki $dW_{\text{маг}} = d\left(\frac{1}{2} L_1 I_1^2\right) + d\left(\frac{1}{2} L_2 I_2^2\right) + d\left(L_{12} I_1 I_2\right),$

ya`ni ikki konturda yig`ilgan magnit energiya quyidagiga anqlanadi:

$$W_{маг} = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + L_{12} I_1 I_2. \qquad (1.12)$$

Demak, n kontur uchun esa quyidagicha yozish mumkin ekan:

$$W_{маг} = \frac{1}{2}\sum_{k=1}^{n} L_k I_k^2 + \frac{1}{2}\sum_{k=1}^{n}\sum_{s=1}^{n} L_{ks} I_k I_s \qquad (1.13)$$

(1.13) dagi $\frac{1}{2}$ ko`paytma $L_{ks}=L_{sk}$ ekanligi hisobga oladi va bu para-metr ikki marta qo`shishda ikki marta uchraydi.

SHuning uchun

$$W_{маг} = \frac{1}{2}\sum_{k=1}^{n} I_k (L_k I_k + \sum_{s=1}^{n} L_{ks} I_s)$$

yoki

$$W_{маг} = \frac{1}{2}\sum_{k=1}^{n} (L_k I_k^2 + \sum_{s=1}^{n} L_{ks} I_s I_k). \qquad (1.14a)$$

(1.19) tenglamani hisobga olgan holda quyidagicha yozish mumkin:

$$W_{маг} = \frac{1}{2}\sum_{k=1}^{n} I_k \Psi_k. \qquad (1.14b)$$

Demak, chiziqli konturli sistemalar uchun quyidagiga ega bo`lamiz:

$$W_{маг} = \sum_{k=1}^{n}\int_{0}^{\psi_k} i_k d\psi_k = \frac{1}{2}\sum_{k=1}^{n} I_k \Psi_k. \qquad (1.15)$$

Bu tenglama to`liq energiyani ifodalaydi.